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Technical Note

Buckling and Postbuckling of a Delaminated Composite Beam in Bending

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I. Introduction

BUCKLING-INDUCED propagation of delaminations is considered one of the critical causes of failure of damaged composites. This can arise from manufacturing faults or during service where low-velocity impact damage can result in barely visible sublayer delaminations.

Many researchers have studied buckling induced delamination under compressive loading. Chai et al. [1] developed a one-dimensional mathematical model for delamination growth in composite plates under compression where the propagation strain depended on the buckling strain of the thin sublaminate.

Others have studied the bending of delaminated composite beams. Shan and Pelegri [2] introduced an analytical model to study the global buckling of a beam with a central delamination and the local buckling of this delamination under axial compression. Murphy and Nichols [3] developed a low-dimensional model to predict the critical moment at which buckling of a thin sublaminate delamination within a cantilever beam occurs under tip shear force. Kardomateas [4] predicted the snap-buckling moment, at which delaminations instantaneously switch from closing to opening for a composite beam under pure bending. In his work, he used the Elastica theory [5] to model the thin delaminated layer. He then used the same theory to study the postbuckling behavior for a delaminated beam under compression [6].

A growing need in the aerospace industry is to have quick and efficient analytical tools for physical insight. Such tools are instrumental in the preliminary analysis phase to study damage tolerance of different delamination scenarios. This Note will study snap-buckling and postbuckling behavior of a thin sublaminate in a composite beam under pure bending using a simple, three-degree-of-freedom, Rayleigh–Ritz method. The study will include a lower bound prediction of the critical moment for snap-buckling and postbuckling moment versus displacement distribution. The model will investigate the bifurcation behavior of the sublaminate, which switches from a closing to an opening mode. Experiments are presented to compare with the predicted moment levels of snap-buckling and postbuckling displacement of the top sublaminate.

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II. Static Test Results

Static tests were performed to understand the buckling and propagation behavior of 16-ply, laminated beams with a through-width delamination created by a polytetrafluoroethylene (PTFE) layer of 0.02 mm thickness and 40 mm length placed at the midlength of the beam between the second and third ply ($a = 0.125$). A unidirectional plate made of M21-T800 carbon material was laid up and then cured using an autoclave. The plate was cut into specimens with dimensions of ($220 \times W \times t$ mm); width W and thickness t are given in Table 1. Each specimen had two strain gauges positioned on the center of the top and bottom surfaces to measure the strain at which buckling of the sublaminate occurred. An INSTRON-1332 servo-hydraulic machine with 10 kN load cell was used for loading application. The strain gauges and loading channel from the servo-hydraulic machine were connected to a SPIDER-8 data acquisition system which was attached to a PC. The specimens were then loaded through a four point bending fixture under displacement control. The spacing of the inner and outer supports was 80 and 120 mm, respectively.

Strain gauges were used to observe the change in the strain value on the top and bottom surfaces of the specimen during buckling. The change in strain was found to occur suddenly in some specimens and with smooth transition for others. Table 1 shows the snap-buckling moment values for different tested specimens. The moment values for the snap buckling had an average value of 268 nm/m, while one test showed a higher moment value of 668 nm/m.

III. Analytical Modelling

A predeveloped mathematical model by Hunt et al. [7], which used a four-degree-of-freedom Rayleigh–Ritz approach to analyze the buckling and postbuckling of a delaminated composite strut under axial compression, was adapted for bending. The model assumes that the beam is isotropic and the undelaminated end laminates are ignored in the prediction of the snap-buckling moment, while the delaminated parts are modeled with bending and in-plane stiffness. Further assumptions are that rotations of the undelaminated region and each delaminated part are the same at their intersection, and that there is no relative shearing movement between laminates at the interface. These rotations form one of the degrees of freedom Q_2. A second degree of freedom represents the end-shortening Δ of the sublaminates over the delaminated region. A further degree of freedom is used to describe the buckling displacement of the delaminated parts Q_1. Figure 1 shows a schematic of the delaminated beam and the applied moment. Note that positive Q_1 and negative Q_2 produce opening.

Table 1 Snap-buckling moments per unit width for tested specimens

Test no.	Width W, mm	Thickness t, mm[a]	Snap-buckling moment, Nm/m[b]
1	8.38	4.22	668
2	8.41	4.31	273
3	8.14	4.17	233
4	9.15	4.22	273
5	10.98	4.27	267
6	10.78	4.30	320
7	10.95	4.24	298
8	10.94	4.33	261
9	11.00	4.22	221

[a]Average 4.25 mm, coefficient of variation (CV) 1%.
[b]Average 268 Nm/m, CV 12%.

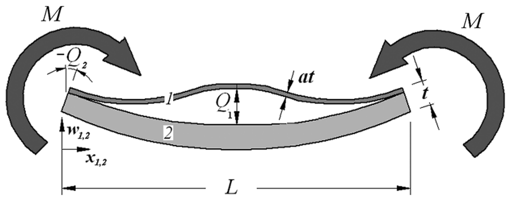

Fig. 1 Delaminated beam coordinates and variables.

The displacement functions for the delaminated and base parts are given as follows:

$$w_1 = Q_1 \sin^2\left(\frac{\pi x_1}{L}\right) + Q_2 \frac{1}{L} x_1 (L - x_1) \tag{1}$$

$$w_2 = \frac{Q_2 x_2 (L - x_2)}{L} \tag{2}$$

The effect of initial imperfection was introduced to the model to investigate how the inclusion of an idealized PTFE layer within the composite layers would affect snap buckling. The imperfection is assumed to have a sine squared function of the form

$$w_i = Q_o \sin^2\left(\frac{\pi x_1}{L}\right) \tag{3}$$

To calculate the stretching energies in the system, the axial end shortening for the thin and thick sublaminates is as follows:

$$\delta_1 = \Delta - \frac{1}{2}\int_0^L \left(\frac{\mathrm{d}(w_1 - w_i)}{\mathrm{d}x_2}\right)^2 \mathrm{d}x_1 - Q_2(1 - a)t \tag{4}$$

$$\delta_2 = \Delta - \frac{1}{2}\int_0^L \left(\frac{\mathrm{d}w_2}{\mathrm{d}x_2}\right)^2 \mathrm{d}x_2 + atQ_2 \tag{5}$$

The strain energy of the system is obtained in terms of bending (U_B) and stretching (U_S) energy as follows:

$$U_B = \frac{EI_1}{2}\int_0^L \left(\frac{\mathrm{d}^2(w_1 - w_i)}{\mathrm{d}x_1^2}\right)^2 \mathrm{d}x_1 + \frac{EI_2}{2}\int_0^L \left(\frac{\mathrm{d}^2 w_2}{\mathrm{d}x_2^2}\right)^2 \mathrm{d}x_2 \tag{6}$$

$$U_S = \frac{t}{2L}(Ea\delta_1^2 + E(1 - a)\delta_2^2) \tag{7}$$

where Δ, E, I are the total end shortening of the sublaminate ends over the delaminated region, Young's modulus and the second moment of the section area, respectively.

The total potential energy of the system is

$$V = U_B + U_S - 2MQ_2 \tag{8}$$

Substituting the displacement equations into the former equation leads to a potential function including three degrees of freedom in addition to the loading moment, i.e., $V(Q_1, Q_2, \Delta, M)$.

By solving the partial equilibrium equation $\partial V(Q_1, Q_2, \Delta, M)/\partial\Delta = 0$ with respect to Δ and substituting the resulting term into V, we obtain a two-degree-of-freedom term $V(Q_1, Q_2, M)$. Solving the adapted system of equations using the methodology reported previously [8], the buckling displacement Q_1 is evaluated as functions of the applied bending moment (M). The derivation process was carried out with algebraic manipulation software.

IV. Analytical Model Results

The model was used to predict the snap-buckling moment for the delaminated beam manufactured from 16 layers of unidirectional (0_{16}) M21-T800 carbon fiber prepreg presented previously. The beam was assumed to have an average thickness of 4.22 mm and a central through-width delamination of 40 mm length situated between the second and third layers. By solving $\partial V/\partial Q_1 = \partial V/\partial Q_2 = 0$ via the system of Eqs. (1–8), the snap-buckling amplitude Q_1 is obtained as a function of the applied bending moment M, using an initial imperfection value $Q_o = 0.02$ mm in Eq. (3). Within the analysis an average of the tensile and compressive modulus was incorporated as the model assumes the material to be isotropic. The average modulus used was 155 GPa. The resulting solution for Q_1 is shown in Fig. 2a. Note that the infeasible solution corresponds to the thin and thick sublaminates passing through each other.

As seen in Fig. 2a, the analytical critical moment does not appear as a distinct point. The critical moment for an unstable system is characterized by more rapid growth of the deflections as the critical moment of the perfect system is approached [7]. To obtain this point, a parametric study was carried out on the system of equations by changing the initial imperfection value (Q_o) to produce the convergence trend of the resulting (Q_1) plots, (see Fig. 2b). For the aforementioned delaminated beam system, the imperfection was changed from 0.02 to 0.07 mm, which showed that the critical moment level converged around a value of 300 Nm/m corresponding to an imperfection value of 0.05 mm. Such difference is attributed to that the assumed imperfection is a squared sine

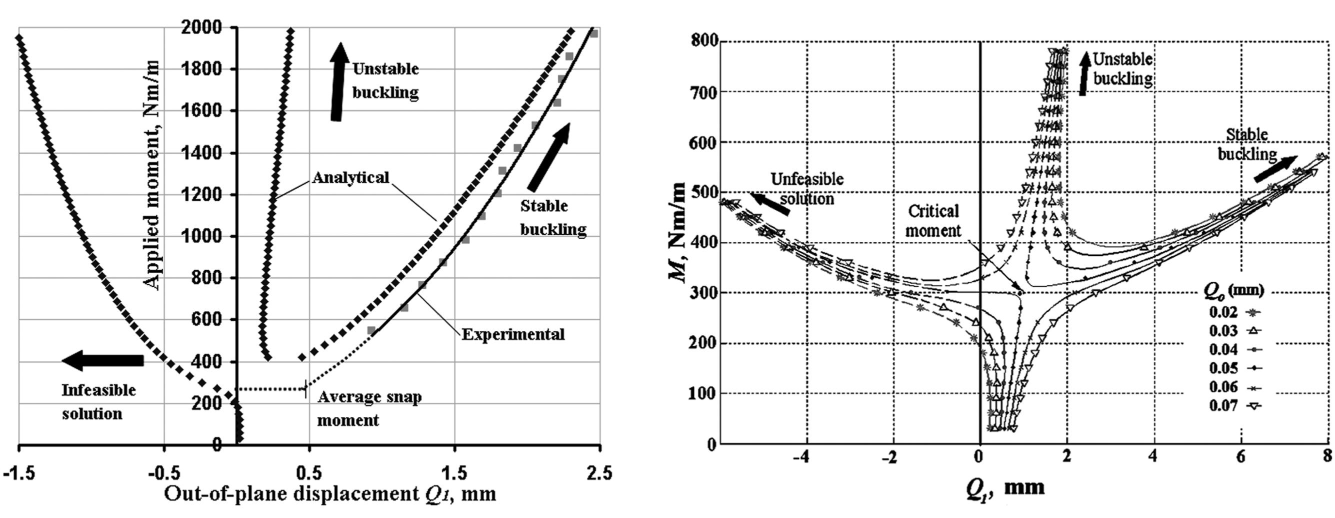

Fig. 2 Displacement Q_1 against applied moment per unit width M: a) experimental and analytical results using $Q_o = 0.02$ mm and b) analytical results for a variety of initial imperfection values, Q_o.

function, see Eq. (3) whereas the real PTFE profile has a flat-stepped shape.

The difference between the experimental and analytical results for Q_1 might be due to the average value used for modulus within the analysis. The modulus of the compressed sublaminate will actually be lower and so buckling would be expected to occur at lower load.

V. Conclusions

The experiments performed on the carbon fiber beams showed an average snap-buckling moment value of 268 Nm/m, which is 12% less than the value obtained using Rayleigh–Ritz methodology. Such analysis included the effect of an initial imperfection on the buckling moment value. The solution of the system revealed three equilibrium branches, see Fig. 2a: infeasible, unstable and stable, where the infeasible branch means the sublaminates pass through one another. One tested specimen with snap-buckling moment value of 668 Nm/m seemed to follow the unstable branch until it snapped to the stable (opening) branch of the plot. Other specimens followed the stable branch of the plot throughout. The effect of the initial imperfection created by the PTFE used to delaminate the beam was included. The analysis has the advantage of being quickly implemented using simple algebraic coding giving engineering insight and providing the basis for future assessment of strength via propagation modelling of the delamination.

Acknowledgments

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References

[1] Chai, H., Babcock, C. D., and Knauss, W., "One Dimensional Modelling of Failure in Laminated Plates by Delamination Buckling," *International Journal of Solids and Structures*, Vol. 17, No. 11, 1981, pp. 1069–1083.
doi:10.1016/0020-7683(81)90014-7

[2] Shan, B., and Pelegri, A., "Approximate Analysis of the Buckling Behavior of Composites with Delamination," *Journal of Composite Materials*, Vol. 37, No. 8, 2003, pp. 673–685.

[3] Murphy, K. D., and Nichols, J. M., "A Low-Dimensional Model for Delamination in Composite Structures: Theory and Experiment," *International Journal of Non-Linear Mechanics*, Vol. 44, No. 1, 2009, pp. 13–18.

[4] Kardomateas, G. A., "Snap Buckling of Delaminated Composites Under Pure Bending," *Composites Science and Technology*, Vol. 39, No. 1, 1990, pp. 63–74.
doi:10.1016/0266-3538(90)90033-2

[5] Britvek, S. J., *The Stability of Elastic Systems*, Pergamon, New York, 1973, pp. 138–152.

[6] Kardomateas, G. A., "The Initial Post-Buckling and Growth Behavior of Internal Delaminations in Composite Plates," *Journal of Applied Mechanics*, Vol. 60, No. 4, 1993, pp. 903–910.
doi:10.1115/1.2901000

[7] Hunt, G. W., Hu, B., Butler, R., Almond, D. P., and Wright, J. E., "Nonlinear Modeling of Delaminated Struts," *AIAA Journal*, Vol. 42, No. 11, 2004, pp. 2364–2372.
doi:10.2514/1.5981

[8] Thompson, J. M. T., and Hunt, G. W., *A General Theory of Elastic Stability*, Wiley, London, 1973, pp. 179–187.

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